Lecture 19: Maximum, minimum values, 2nd derivative, concavity, L'Hospital's Rule

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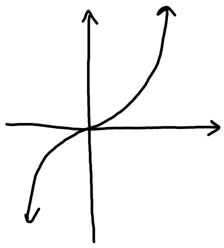
f' is
- the first derivative
- the slope of the tangent

If:
$f'(x) > 0$: f is increasing
$f'(x) < 0$: f is decreasing

NOTE: Sometimes $f'(c) = 0$, but c isn't a minimum or a maximum.
 Ex. $f(x) = x^3, f'(x) = 3x^2$, which is 0 at $x = 0$:



$f(x) = x^3$ is strictly growing on $\mathbb{R}$

If $f'(c) = 0$ for some point c, then check $f''(c)$:
- If $f''(c) > 0$: concave up, local min
- If $f''(c) < 0$: concave down, local max
- If $f''(c) = 0$: no curvature

- If $f''(c)$ changes sign around c, then c is called an **inflection point**.

Example
f'
Where is $x^3 - 7x^2 - 2x + 4 = f(x)$ increasing, decreasing,
concave upwards and concave downwards?
f''

$f'(x) = 3x^2 - 14x - 2$
$f''(x) = 6x - 14$

$f'(x) = 0$ at $x_1 = -0.1387, x_2 = 4.8054$

$f'$is:
positive for $x < -0.1387$
negative for $x > 4.8084 \rightarrow$ increasing
deceasing on $-0.1087 < x < 4.8054$

$f''(x) = 0$ at $x = \frac{7}{3}$
To determine where negative/positive, use $f''(0) = \underline{-14}, f''(3) = 18 - 14 = \underline{4}$

change of sign!

inflection point!

Applications of derivatives to limits (L'Hospital's Rule)

If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$ then:

$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

$\to$ a is a number, ∞, or $-\infty$.

<u>Examples</u>

(1)

$$\lim_{x\to 0}\frac{\sin(x)}{x} \overset{0/0}{=} \lim_{x\to 0}\frac{(\sin(x))'}{(x)'} = \lim_{x\to 0}\frac{\cos(x)}{1} = \frac{1}{1} = 1$$

(2)

$$\lim_{x\to 0}\frac{e^x-1}{\sin(x)} \overset{0/0}{=} \lim_{x\to 0}\frac{e^x}{\cos(x)} = \frac{1}{1} = 1$$

(3)

$$\lim_{x\to\infty}\frac{\ln(x)}{x^2} = \frac{\infty}{\infty} \quad \to L'Hospital$$

$$= \lim_{x\to\infty}\frac{\frac{1}{x}}{2x} = \lim_{x\to\infty}\frac{1}{2x\cdot x} = 0$$

(4)

$$\lim_{x\to\infty}\frac{\mathrm{x}^2}{e^x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty}\frac{2x}{e^x} = \frac{\infty}{\infty} \quad \text{(apply again)}$$

$$= \lim_{x\to\infty}\frac{2}{e^x} = 0$$

POWERS

$$= \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x \qquad \backslash\ e^{\ln(a)} = a$$

$$= \mathrm{e}^{\ln\left(\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x\right)}$$

$$= \mathrm{e}^{\lim_{x\to\infty}\ln\left(\left(1-\frac{1}{x}\right)^x\right)}$$

... finish on your own ...

<u>Final answer</u>:

$= -1$